

Demand Economics*

The ... "*successful operation of any economic organization requires a thorough understanding of demand and supply conditions for its products.*"

Economic demand refers to the amount of a product that people are willing and able to buy under a given set of conditions.

Note: need or desire is a necessary component but must be accompanied by financial capability before an economic demand is created.

History: Leon Walras (1834-1910); Alfred Marshall (1842-1924); Vilfredo Pareto (1848-1923); Eugen Slutsky (1880-1948); Kenneth Arrow (1921-) and Gerard Debreu (1921-).

Economic supply is the amount of a good or service that firms will make available for sale under a given set of conditions. Note: supply requires a desire to sell along with the economic capability to bring a product to market.

When we bring demand and supply together we create a framework for analyzing the interaction of buyers and sellers.

Wikipedia: http://en.wikipedia.org/wiki/Supply_and_demand

*This presentation assumes the student has completed an introductory course in micro-economics.

THE BASIS FOR DEMAND

Demand is the quantity of a good or service that customers are willing and able to purchase during a specified period under a given set of conditions:

- time frame { day, hour, etc. }
- conditions { price of good, consumer incomes }

Managerial economists focus on market demand.

- direct demand (theory of consumer behavior)
- derived demand (inputs used in production)

INDUSTRY DEMAND vs FIRM DEMAND

1. Market demand functions can be specified for an entire industry or for an individual firm.
2. Might use different variables:
 - a. firm demand is negatively related to its own price
 - b. firm demand may be positively related to competitors price.
 - c. firm demand would typically increase with firm advertising
 - d. firm demand might decrease with increase in adv by competitor.

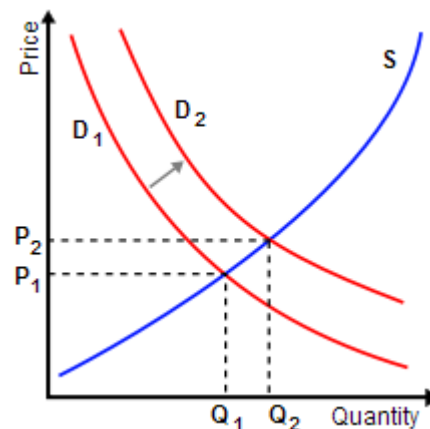
Focus: THE DEMAND CURVE

The demand **curve** is the part of the demand function that expresses the relationship between the price charged for a product and the quantity demanded.

Relation Between Demand Curve and Demand Function

1. **Change in Quantity Demanded** -- movement along a given demand curve. A change in quantity demanded refers to the effect on sales of a change in price, holding constant the effects of all other demand-determining factors.
2. **Shift in Demand** -- a shift from one demand curve to another, reflects a change in one or more of the nonprice variables in the product demand function.

The task of demand analysis is distinguish between changes in the quantity demanded (movements along a given demand curve) and changes in demand (shifts from one demand curve to another). The process is complicated by the fact that not only prices, but also income, population, interest rates, advertising, etc. vary from period to period.



Source: Wikipedia

A shift in the demand curve means that either more or less will be demanded at each and every ruling price in the market. Essentially - shifts in demand are caused by changes in the willingness and ability of consumers to buy a particular product at a given price.

- i) Changing price of a substitute -- *Substitutes are goods in competitive demand and act as replacements for another product.*
- ii) Changing price of a complement -- *A complement tends to be bought together with another good (fish and chips). A rise in the price of a complement to Good X should cause a fall in the demand for X.*
- iii) Change in the income of consumers
- iv) Change in tastes and preferences
- v) Changes in interest rates

Exceptions to the law of demand

Giffen Goods: These are highly inferior goods that people on low incomes spend a high proportion of their income on. When price falls, they are able to discard the consumption of these goods (having already satisfied their demand) and move onto better goods. Demand may fall when the price falls. These tend to be very basic foods such as rice and potatoes.

Ostentatious Consumption: Some goods are luxurious items where satisfaction comes from knowing the price of the good. A higher price may be a reflection of quality and people on high incomes are prepared to pay this for the "snob value effect" Examples would include perfumes, designer clothes, fast cars.

THE BASIS FOR SUPPLY

Supply refers to the quantity of a good or service that producers are willing and able to sell during a specific period and under a given set of conditions.

-- conditions (price, price of related goods, technology, etc).

The supply of a product in the market is merely the aggregate of the amounts supplied by individual firms.

The Market Supply Function

The market supply function is a statement of the relation between the quantity supplied and all factors affecting the quantity.

$$Q_x = f (P_x, P_y, P_c, P_l, P_i \quad \} \text{ Price Variables} \\ T, \quad \} \text{ Technology} \\ G, N, W) \quad \} \text{ Other}$$

Where:

Q_x	=	Quantity Demanded
P_x	=	Price of Product x
P_y	=	Price of Product y
P_c	=	Price of Capital
P_l	=	Price of Labor
P_i	=	Interest cost
T	=	Technology / Innovation
G	=	Government Policy
N	=	Number of People in the Economy
W	=	Weather Conditions

TOPICS -- SUPPLY CURVE & FUNCTION

1. Industry Supply versus Firm Supply
2. Supply Curve - price and supply holding all else constant.
3. Change in quantity supplied -- a movement along a given supply curve.
4. Shift in supply -- a movement from one supply curve to another.
5. Comparative Statics – The study of changing demand and supply conditions. For example, examining how demand varies with changing interest rates while holding supply conditions constant; or, holding demand constant, investigating how supply will change with varying interest rates.

EQUILIBRIUM

When the quantity demanded and the quantity supplied of a product are in perfect balance at a given price, the market is said to be in equilibrium.

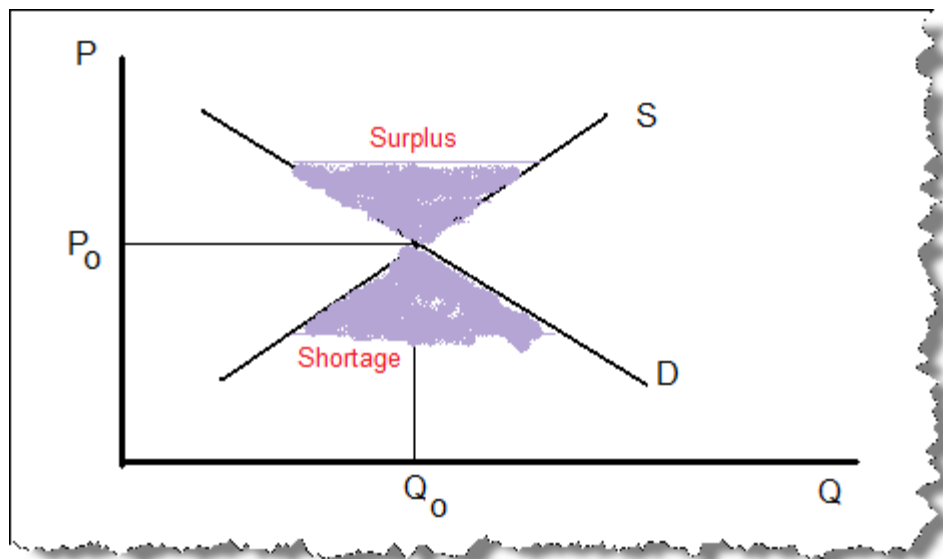
An equilibrium is stable when the factors underlying demand and supply conditions remain unchanged in both the present and the foreseeable future.

Surplus

A surplus is created when producers supply more of a product at a given price than buyers demand (excess supply).

Shortage

A shortage is created when buyers demand more of a product at a given price than producers are willing to supply (excess demand).



DEMAND ANALYSIS:

OPTIMAL PRICING AND ELASTICITY

The demand curve is a special subcase of the demand function in which *ceteris paribus* (all else held constant) applies to all independent variables except the price of the product in question. Since none of the other independent variables or the residual term ε vary when *ceteris paribus* is in force, it is possible to compress them all into a single term A and express the demand function as follows:

$$Q_x = A + \beta_1 P_x + \varepsilon$$

where the parameter A includes the influence of all the other independent variables and the residual error term ε .

NOTE: the demand curve expresses the relationship between Q_x and P_x with all other things remaining constant. Many economists follow the convention set by Alfred Marshall (the great classical economist) and traditionally place the independent variable (price) on the vertical axis for their graphical analysis. Thus, we may often see the demand curve in this form:

$$P_x = a + bQ_x$$

Let's not forget, however, that P_x is the independent variable and Q_x the dependent one.

To obtain P_x from Q_x :

1. Subtract A from both sides
2. Divide both sides by β_1

$$P_x = \frac{-A}{\beta_1} + \frac{1}{\beta_1} Q_x$$

Letting $a = \frac{-A}{\beta_1}$ and $b = \frac{1}{\beta_1}$

$$P_x = a + b Q_x$$

Note that the numerical value of β_1 is expected to have a negative sign because of the law of demand. Thus the parameter a will be a positive number, and b will be a negative number.

Numerical Example

Suppose the demand function for product X has been estimated using regression analysis as follows (also, see example at end):

$$Q_X = 5,030 - 3,806.2P_X + 1,458.5P_Y + 256.6A_X - 32.3A_Y + 0.18Y_C$$

Assume the following values for the independent variables:

$$P_X = \$8$$

$$P_Y = \$6$$

$$A_X = \$168(\text{in thousands})$$

$$A_Y = \$182(\text{in thousands})$$

$$Y_C = \$12,875$$

Substituting the values into the estimated equation:

$$Q_X = 5,030 - 30,449.6 + 8,751.0 + 43,108.8 - 5,878.6 + 2,317.5$$

for example, (8)(-3806.2) = -30,449.6

Thus, it possible to predict quantity demanded of product X should be 22,879.1 units, give ceteris paribus.

Further, where the demand curve is concerned, note that if we sum 5,030 which is the constant term, A, and the influence of all other independent variables except P_X then

$$Q_X = 53,328.7 - 3,806.2P_X$$

For convenience, divide through by 1,000

$$Q_x = 53.3287 - 3.8062P_x$$

To invert this expression to a specification of P_x (see above)

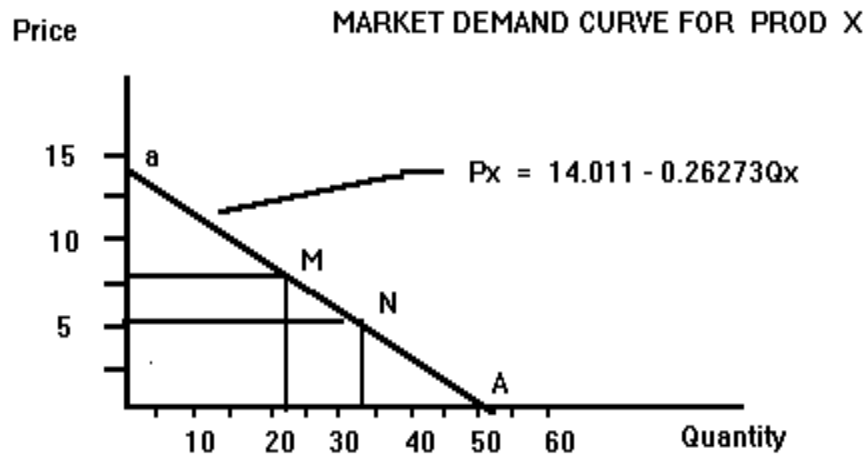
1. add $3.8062P_x$ to both sides
2. subtract Q_x from both sides
3. divide both sides by $3.8062P_x$

This yields the following demand curve from the estimated demand function:

$$P_x = 14.011 - 0.26273Q_x$$

Findings:

1. price is expressed as a linear function of quantity demanded, *ceteris paribus*.
2. a is the intercept term on the vertical (price) axis
3. b is the slope term
4. curve intercepts the price axis at the value a , and slopes downward at the rate b , or $1/\beta_1$.
5. The intercept on the horizontal axis is the value A since this is the value of Q_x when P_x is zero.



RELATIONSHIP AMONG: PRICE, TR, AND MR

Why are we interested in the variation between price and quantity of a particular commodity?

Our concern is to see what happens to total sales revenue when prices and quantities are varied. Remember, no matter what sub-objectives decision makers hold, *total revenue* is likely to play a major role in the optimization of cash flow management, and hence, achieving optimal capital investment budgets.

Def: Total Revenue = $TR_x = P_x \cdot Q_x$
given,

$$P_x = a + b Q_x$$

substitute P_x into total revenue equation, for

$$TR_x = a Q_x + b Q_x^2$$

DEF: Marginal Revenue is the change in total revenue that results from a one-unit increase in quantity demanded.

Since MR is defined as the change in total revenue for a one-unit change in quantity demanded, it can be expressed as the first derivative with respect to Q_x .

$$MR_x = a + 2bQ_x$$

A COMPARISON WITH DEMAND CURVE

1. intercept is a .
2. the slope of MR curve is twice that of the demand curve:

$$P_x = 14.011 - 0.26273Q_x , \text{ and}$$

$$MR_x = 14.011 - 0.52546Q_x$$

These relationships can be summarized in a single number know as "*price elasticity*" of demand.

Elasticity is used to express the responsiveness of one variable to a change in another variable.

DEF: Stated more rigorously, an elasticity is the percentage change in the dependent variable occasioned by a 1 *percent change* in an independent variable.

PRICE ELASTICITY OF DEMAND

DEF: the percentage change in quantity demanded divided by the percentage change in price which caused the change in quantity demanded.

<p style="text-align: center;">Computing the Price Elasticity of Demand</p> <p><i>Price elasticity of demand = $\frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}}$</i></p> <p>Example: If the price of an ice cream cone increases from \$2.00 to \$2.20 and the amount you buy falls from 10 to 8 cones then your elasticity of demand would be calculated as:</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\frac{\frac{(10 - 8)}{10} \times 100}{\frac{(2.20 - 2.00)}{2.00} \times 100} = \frac{20 \text{ percent}}{10 \text{ percent}} = 2$ </div>	$\epsilon = \frac{\% \text{ change in } Q_x}{\% \text{ change in } P_x}$
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$$\frac{\frac{\Delta Q_x}{Q_x} \cdot \frac{100}{1}}{\frac{\Delta P_x}{P_x} \cdot \frac{100}{1}} \quad ; \text{-or-}$$

$$\epsilon = \frac{\Delta Q_x}{Q_x} \cdot \frac{P_x}{\Delta P_x}$$

Arc / Discrete Elasticity	Point (Continuous) Elasticity
$\epsilon = \frac{\Delta Q_x}{\Delta P_x} \cdot \frac{P_x}{Q_x}$	$\epsilon = \frac{dQ_x}{dP_x} \cdot \frac{P_x}{Q_x}$

Arc / Discrete Elasticity

Computing the Price Elasticity of Demand Using the Midpoint Formula

The **midpoint formula** is preferable when calculating the price elasticity of demand because it gives the same answer regardless of the direction of the change.

$$\text{Price Elasticity of Demand} = \frac{(Q_2 - Q_1) / [(Q_2 + Q_1) / 2]}{(P_2 - P_1) / [(P_2 + P_1) / 2]}$$

Computing the Price Elasticity of Demand

$$\text{Price Elasticity of Demand} = \frac{(Q_2 - Q_1) / [(Q_2 + Q_1) / 2]}{(P_2 - P_1) / [(P_2 + P_1) / 2]}$$

Example: If the price of an ice cream cone increases from \$2.00 to \$2.20 and the amount you buy falls from 10 to 8 cones the your elasticity of demand, using the **midpoint formula**, would be calculated as:

$$\frac{\frac{(10 - 8)}{(10 + 8) / 2}}{\frac{(2.20 - 2.00)}{(2.00 + 2.20) / 2}} = \frac{22 \text{ percent}}{9.5 \text{ percent}} = 2.32$$

RELATIONSHIP BETWEEN PRICE ELASTICITY AND TOTAL REVENUE

	Price Elasticity Demand (PED)	Price ↑	Price ↓
Elastic	$\infty > \epsilon > 1$	TR ↓	TR ↑ $\% \Delta Q > \% \Delta P$
Unitary	$\infty > \epsilon = 1$	TR →	TR → $\% \Delta P = \% \Delta Q$
Inelastic	$0 < \epsilon < 1$	TR ↑	TR ↓ $\% \Delta P > \% \Delta Q$

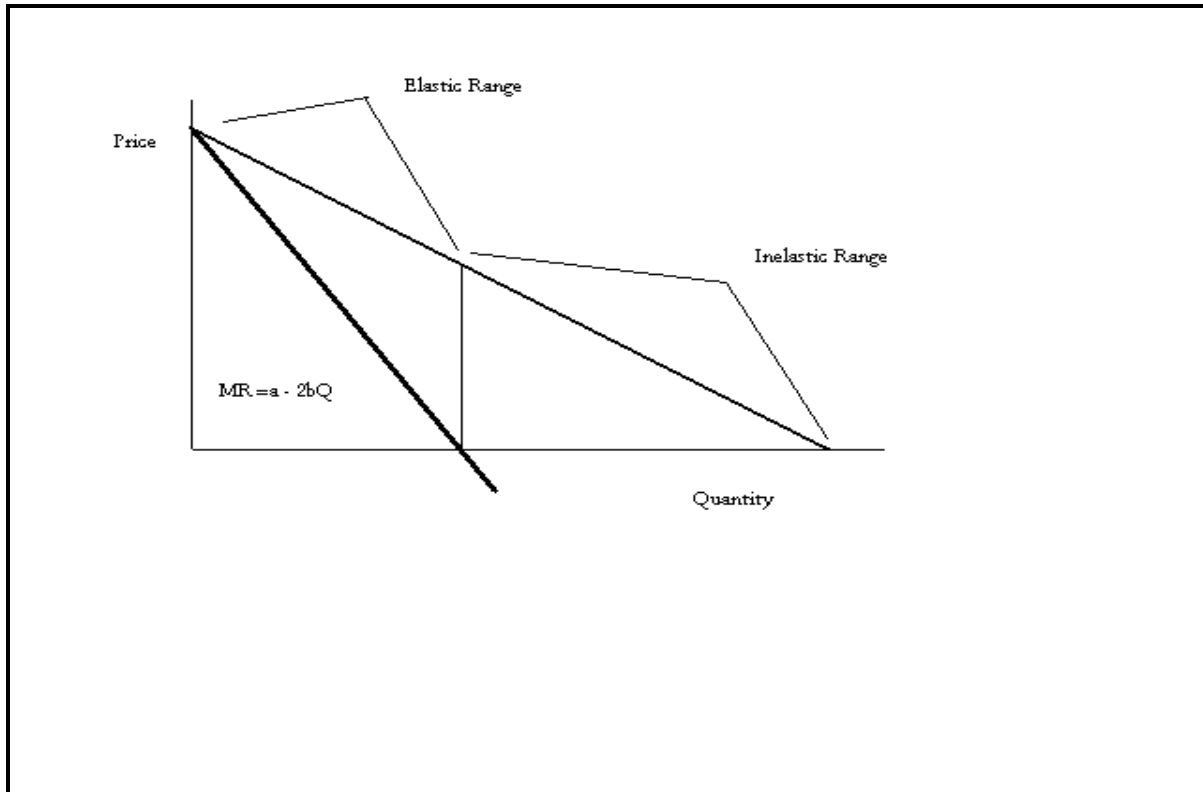
Ranges of Elasticity

Inelastic Demand

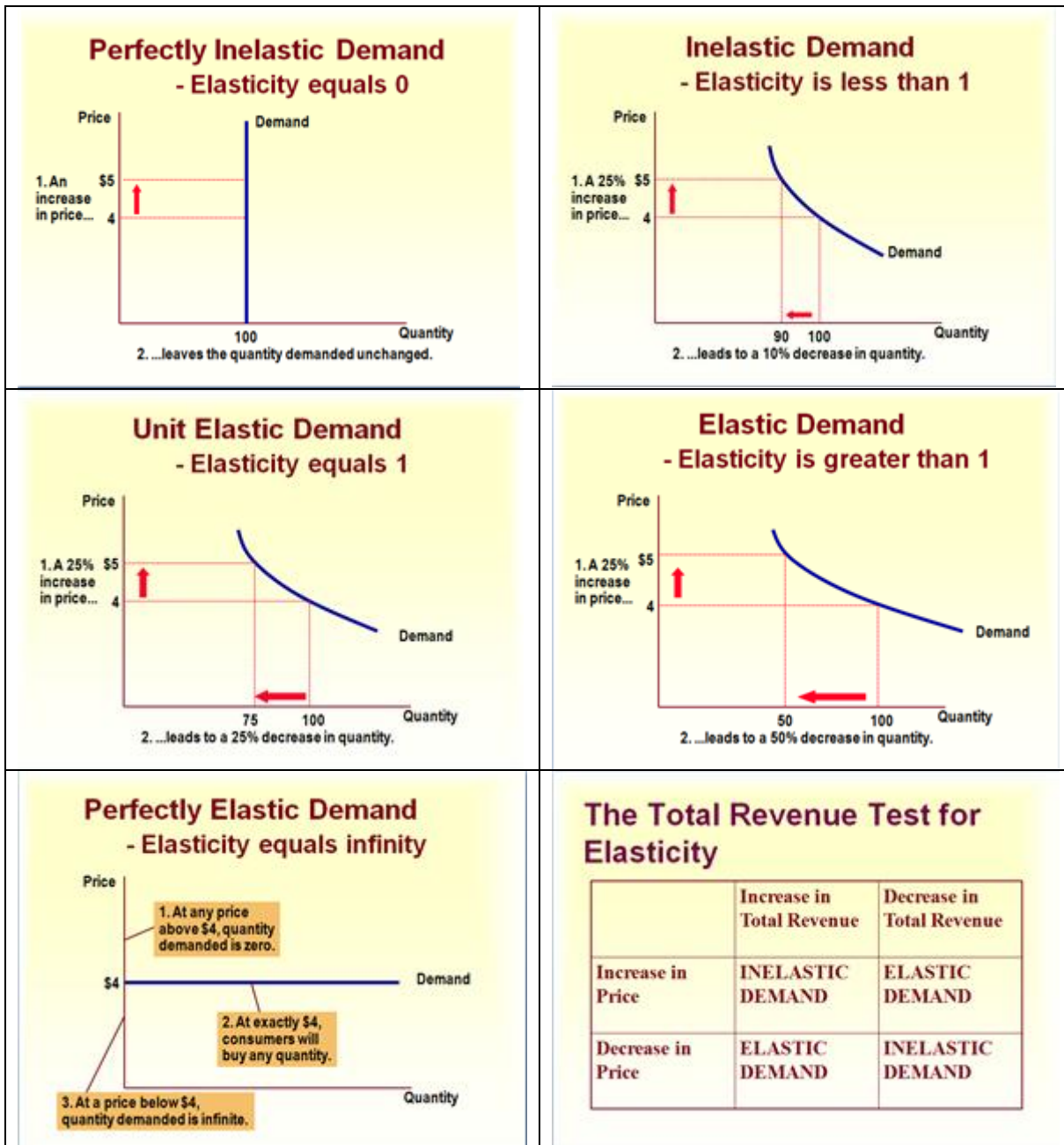
- ◆ Percentage change in price is greater than percentage change in quantity demand.
- ◆ Price elasticity of demand is *less than one*.

Elastic Demand

- ◆ Percentage change in quantity demand is greater than percentage change in price.
- ◆ Price elasticity of demand is *greater than one*.



Note: When MR is positive, PED is elastic; and, when negative, PED is inelastic.



IMPLICATIONS FOR OPTIMAL PRICES (P*)

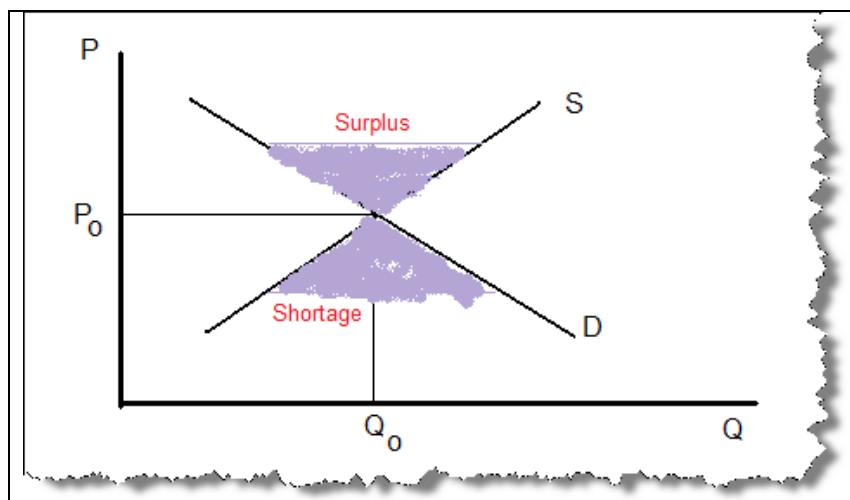
(a) Total Revenue = $TR_x = P_x \cdot Q_x$

(b) Marg. Revenue = $\frac{dTR}{dQ} = \frac{d(P_x \cdot Q_x)}{dQ_x}$

= $P \cdot \frac{dQ}{dQ} + Q \cdot \frac{dP}{dQ}$; = $P \cdot 1 + Q \cdot \frac{dP}{dQ}$

= $P + Q \cdot \frac{dP}{dQ}$; MR = $P \left(1 + \frac{Q}{P} \cdot \frac{dP}{dQ} \right)$

<p>NOTE: $\frac{Q}{P} \cdot \frac{dP}{dQ}$ is simply</p> $\frac{1}{\frac{dQ}{dP} \cdot \frac{P}{Q}} = \frac{1}{\epsilon_p}$ <p>MR = $P \left(1 + \frac{1}{\epsilon_p} \right)$</p>	<p>Optimal Price (P*) always occurs where MC = MR</p> <p>MC = $P \left(1 + \frac{1}{\epsilon_p} \right)$, or</p> $P^* = \frac{MC}{1 + \frac{1}{\epsilon_p}}$
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For a firm facing perfectly competitive markets, price does not change with quantity sold, $\frac{dP}{dQ} = 0$, in which case marginal revenue is equal to price.

By contrast, for a monopoly, the price received will decline with quantity sold, $\frac{dP}{dQ} < 0$, so that marginal revenue is less than price.

Implications: the profit-maximizing quantity (MR=MC) for a monopoly will be lower than that for a competitive firm while the corresponding profit maximizing price will be higher.

Summary: Uses Of Price Elasticity

1. Examine the effect of price increases on total revenue
2. To determine how great a price reduction is needed to increase sales by some percentage (eg., 10%)
3. Determine the profit maximizing price

INCOME ELASTICITY

An important determinant of demand. Luxuries and big-ticket items for sure. Use either a *per capita*, *household*, or *aggregate* basis.

Income Elasticity: the responsiveness of demand to changes in income, holding constant the effect of all other variables. Let I represent the consumers budget:

$$\varepsilon_I = \frac{\% \text{ change in } Q_x}{\% \text{ change in } I}$$

Income and quantity purchased typically move in the same direction; income and sales are directly related.

Def: Luxuries -- products for which the proportionate change in quantity demanded is greater than the proportionate change in consumer income levels. Examples include: fur coats, travel by air, etc.

Def: Necessities or normal goods -- products which have a positive income elasticity of demand. Demand rises as the national economy expands and incomes increase.

Def: Inferior goods -- products which exhibit a negative (*inverse*) income effect and, consequently, an income elasticity which is negative. These products experience a decline in quantity demanded as real income levels rise. As incomes rise, people tend to switch away from these items to more desirable substitutes. As income fall, people (reluctantly) switch back to cheaper alternatives and away from more desirable (but more expensive) substitutes.

- a. non-cyclical normal: $0 < \epsilon_I < 1$ Convenience items
toiletries, movies
- b. cyclical normal: $\epsilon_I > 1$ Luxuries see above
- c. Inferior (counter cyclical): $\epsilon_I < 0$ Basic foodstuffs,
generics

Alternative Classification

I Elasticity	Classification	Income ↑	Income ↓
$\infty > \epsilon_I > 1$	Luxuries	Q ↑/gr %	Q ↓/gr %
$1 > \epsilon_I = 0$	Necessities	Q ↑/ls %	Q ↓/ls %
$0 > \epsilon_I > -\infty$	Inferior	Q ↓	Q ↑

CROSS (Price) ELASTICITY

Def: the responsiveness of demand for one product to changes in the price of another

$$\epsilon_{px} = \frac{\% \text{ change in } Q_x}{\% \text{ change in } P_y}$$

Def: the percentage change in quantity demanded of product X, divided by the percentage change in the price of some other product, Y .

Substitutes: products among which the cross elasticity of demand is positive. When the price of product Y is reduced the quantity demanded of product X is reduced.

This is because the price of product Y enters the demand function as a shift parameter; a reduction in the price of product Y would cause the demand curve for product X to shift to the left, such that a price P_x the quantity demanded for product X would be somewhat less.

Examples include -- tea and coffee; Maxwell House coffee and Folger's Coffee.

Complements -- products among which the cross elasticity of demand is negative. Price and quantity move in opposite directions for complementary goods and services. A price increase in one product typically leads to a reduction in demand for the other.

Products which are used jointly in consumption and in some predetermined ratio. Examples include -- beer and pretzels, gasoline and tires, cameras and film, or computers and software, etc.

ϵ_{px} - Elasticity /Relationship	$P_y \uparrow$	$P_y \downarrow$
$\infty > \epsilon_{px} > 1$ Substitutes	Q_x rises	Q_x falls
$\epsilon_{px} = 0$ Unrelated	Q_x no chg	Q_x no chg
$0 > \epsilon_{px} > -\infty$ Complements	Q_x falls	Q_x rises

CROSS ADVERTISING ELASTICITY

The advertising elasticity of demand for product X measures the responsiveness of the change in quantity demanded to a change in advertising expended for product X. Expect a positive relationship.

Cross-advertising elasticity measures the responsiveness of quantity demanded (sales) of product X to a change in advertising efforts directed at another product Y . Expect the relationship to be negative between substitute products and positive between complementary products.

For example, increased advertising efforts for a particular movie would be expected to reduce the quantity demanded of admission tickets to other movies and attractions but to increase the sales of the refreshment stand in that particular theater.

Example: MMH, Problem 7-3

Actual Data

Observation	Sales*	Advertising	Price
	Dep	Ind	Ind
1.000	495.000	900.000	150.000
2.000	555.000	1200.000	180.000
3.000	465.000	750.000	135.000
4.000	675.000	1350.000	135.000
5.000	360.000	600.000	120.000
6.000	405.000	600.000	120.000
7.000	735.000	1500.000	150.000
8.000	435.000	750.000	150.000
9.000	570.000	1050.000	165.000
10.000	600.000	1200.000	150.000

*note: this should be a quantity variable (Q)

Regression Parameters

Variable	Parameter Estimate (Est)	Standard Error	t For Ho: Est=0.0	P-Value (95%=0.05)	Partial Corr	VIF
Intercept	247.644	62.818	3.942	0.005	0.689	0.000
Advertising	0.393	0.030	13.250	0.000	0.962	1.404
Price	-0.734	0.502	-1.463	0.180	-0.234	1.404

Partial Correlation: measures the degree of association between two random variables, with the effect of all other variables removed.

http://en.wikipedia.org/wiki/Partial_correlation

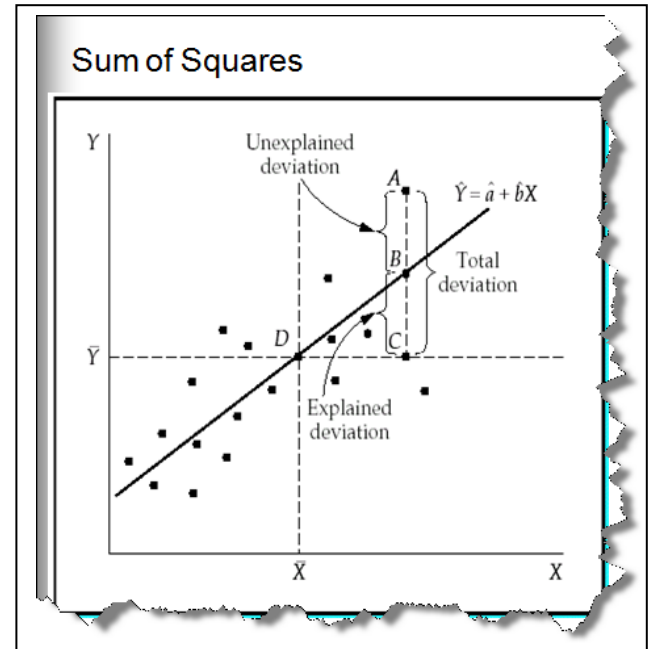
Variance Inflation Factor (VIF): is a method of detecting the severity of multicollinearity. http://en.wikipedia.org/wiki/Variance_inflation_factor

Regression ANOVA Table

Dep: Sales				
Sources	SSQ	MSQ	Df	F-Value
Model	125193.471	62596.735	2.000	110.122
Error	3979.029	568.433	7.000	P-Value
C.Total	129172.500		9.000	0.000
Association Test		MLE Stats		
Root MSE	23.842	Lambda	====>	n/c
SSQ(Res)	3979.029	LogLikelihood	====>	n/c
Dep.Mean	529.500			
Coef of Var (CV)	4.503			
R-Squared	0.969			
Adj R-Squared	0.960			
Auto Correlation		Diagnostic Tests		
Rho	-0.300	White's	====>	5.297
Durbin	2.576	Homoskedasticity	====>	0.381
Durbin H	n/c			
D Low Limit	0.697	Average VIF	====>	1.404
D Upper Limit	1.641			
Ho: Rho = 0				
Rho: Pos & Neg	DoNot Reject			
Rho: Positive	Inconclusive			
Rho: Negative	Inconclusive			

Table of Residuals

Obs	Sales (Quantity)	Sales	Residual
	Actual	Predicted	
Obs 1	495.000	490.867	4.133
Obs 2	555.000	586.619	-31.619
Obs 3	465.000	442.992	22.008
Obs 4	675.000	678.526	-3.526
Obs 5	360.000	395.116	-35.116
Obs 6	405.000	395.116	9.884
Obs 7	735.000	726.402	8.598
Obs 8	435.000	431.984	3.016
Obs 9	570.000	538.743	31.257
Obs 10	600.000	608.635	-8.635



$$Y = \beta_0 + \beta_1 (\text{Advertising}) + \beta_2 (\text{Price})$$

$$\text{Sales} = \beta_0 + \beta_{\text{Adv}} (\text{Advertising}) + \beta_{\text{Price}} (\text{Price})$$

$$\begin{aligned} \text{Predicted Sales} &= 247.644 + 0.393(900) - 0.734(150) = \\ &= 247.644 + 353.7 - 110.1 = 491.244^* \end{aligned}$$

*Round-off :
calculator vs
computer
[490.867]

Computed “Point” Elasticity

	Advertising	Price
	I	I
Obs 1	0.714	-0.222
Obs 2	0.849	-0.238
Obs 3	0.633	-0.213
Obs 4	0.785	-0.147
Obs 5	0.654	-0.245
Obs 6	0.582	-0.217
Obs 7	0.801	-0.150
Obs 8	0.677	-0.253
Obs 9	0.723	-0.212
Obs 10	0.785	-0.183
Average==>	0.720	-0.208

Where point elasticity (assume price elasticity) is:

$$\epsilon = \frac{dQ_x}{dP_x} \cdot \frac{P_x}{Q_x}$$

$$\epsilon = -0.734 * [150 / 495] = -0.222$$

The Multiplicative Demand Model

A generalized model where:

<i>Variable</i>	<i>Description</i>
P	Price
Y	Income
P_{op}	Population
A	Advertising
Q_D	Quantity demanded

$$(1) Q_D = \beta_0 P^{\beta_1} Y^{\beta_2} P_{op}^{\beta_3} A^{\beta_4} \varepsilon$$

Take logs of both sides

$$(2) \ln Q_D = \ln \beta_0 + \beta_1 \ln P + \beta_2 \ln Y + \beta_3 \ln P_{op} + \beta_4 \ln A + \ln \varepsilon$$

Elasticity Concepts

1. Elasticities are constant over the range of the data used in estimating the parameters.
2. The Elasticities are equal to the estimated values of the respective parameters.
3. The property of constant elasticity means that a given percentage change in one of the independent variables, such as income, will result in the same proportionate percentage change in quantity demanded at all points on the demand curve.

For example, the income elasticity of demand is defined as:

$$(3) \quad \varepsilon_Y = \left(\frac{\partial Q_D}{\partial Y} \right) \left(\frac{Y}{Q_D} \right)$$

Differentiating with respect to income (drop the disturbance term):

$$(4) \quad \left(\frac{\partial Q_D}{\partial Y} \right) = \beta_2 \beta_0 P^{\beta_1} Y^{\beta_2-1} P_{op}^{\beta_3} A^{\beta_4}$$

$$(5) \quad \varepsilon_Y = \beta_2 \beta_0 P^{\beta_1} Y^{\beta_2-1} P_{op}^{\beta_3} A^{\beta_4} \cdot \left(\frac{Y}{Q_D} \right)$$

Substituting equation (1) yields:

$$(6) \quad E_Y = \beta_2 \beta_0 P^{\beta_1} Y^{\beta_2-1} P_{op}^{\beta_3} A^{\beta_4} \cdot \frac{Y}{\beta_0 P^{\beta_1} Y^{\beta_2} P_{op}^{\beta_3} A^{\beta_4}}$$

By canceling and combining terms where possible:

$$\varepsilon_Y = \frac{\beta_2 \beta_0 P^{\beta_1} Y^{\beta_2} P_{op}^{\beta_3} A^{\beta_4}}{Y} \cdot \frac{Y}{\beta_0 P^{\beta_1} Y^{\beta_2} P_{op}^{\beta_3} A^{\beta_4}} = \beta_2$$

Linear Vs. Non-linear

1. Depends on the true relationships as closely as possible.
2. Clue: graph dependent variable over time and each independent variable against the dependent variable. The results will suggest whether a linear equation is most appropriate or whether logarithmic, exponential, or other transformations are more appropriate.

Actual Data: Ln Transformation of MMH 7-3

Observation	Sales	Advertising	Price
	D	I	I
1.000	495.000	900.000	150.000
2.000	555.000	1200.000	180.000
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7.000	735.000	1500.000	150.000
8.000	435.000	750.000	150.000
9.000	570.000	1050.000	165.000
10.000	600.000	1200.000	150.000

Ln Transformed Data

Observation	Sales	Advertising	Price
	D	I	I
Obs1	6.205	6.802	5.011
Obs2	6.319	7.090	5.193
Obs3	6.142	6.620	4.905
Obs4	6.515	7.208	4.905
Obs5	5.886	6.397	4.787
Obs6	6.004	6.397	4.787
Obs7	6.600	7.313	5.011
Obs8	6.075	6.620	5.011
Obs9	6.346	6.957	5.106
Obs10	6.397	7.090	5.011

Multiplicative Regression Parameters

Variable	Parameter Estimate	Standard Error	t For Ho: Est = 0	P-Value (95%=0.05)	Partial Corr	VIF
Intercept	2.448	0.616	3.973	0.004	0.693	0.000
Advertising	0.730	0.062	11.829	0.000	0.952	1.659
Price	-0.241	0.158	-1.519	0.165	-0.248	1.659

Dependent: Sales

Estimated elasticity values. Avg. elasticity values (linear model): 0.720 and -0.208, respectively

Multiplicative Regression ANOVA

Dep: Sales					
Sources	SSQ	MSQ	Df	F-Value	P-Value
Model	0.449	0.225	2.000	99.215	
Error	0.016	0.002	7.000		
C.Total	0.465		9.000	0.0001	
Association Test		MLE Stats			
Root MSE	0.048	Lambda	====>	n/c	
SSQ(Res)	0.016	LogLikelihood	====>	n/c	
Dep.Mean	6.249				
Coef of Var (CV)	0.762				
R-Squared	0.966				
Adj R-Squared	0.956				
Auto Correlation		Diagnostic Tests			
(auto corr) Rho	-0.306	White's	====>	6.213	
Durbin	2.591	Homoskedasticity	====>	0.286	
Durbin H	n/c				
D Low Limit	0.697	Average VIF	====>	1.659	
D Upper Limit	1.641				
Ho: Rho = 0					
Rho: Pos & Neg	DoNot Reject				
Rho: Positive	Inconclusive				
Rho: Negative	Inconclusive				

Four Major Assumptions of Linear Regression (OLS)

1. Multicollinearity
 - a. Regression coefficients may be misleading
 - i. Sensitive to small changes / more obs.
 - ii. May have opposite sign to hypothesis
 - iii. Significance may change given mix of variables in model
 - b. VIF – each individual variable (≤ 5 ; < 10)
 - c. VIF – average (Anova tab)
2. Time-Series Models – Absence of Autocorrelation
 - a. Durbin-Watson test (Anova tab)
 - b. Two-tailed test at 95% C.L. (either direction)
 - c. Row 26 Anova tab (Reject, Inconclusive, Do Not Reject)
3. Existence of constant variance (CV)
 - a. Whites' Test (Anova tab)
 - b. Ho is affirmative statement (residuals are homoscedastic)
 - c. Objective is NOT to reject Ho (p-value should be larger than 0.05)
 - d. View CV diagnostic graph. Half the points should be above/below the demarcation line within an equal band.
4. Normality (Anova tab)
 - a. Critical correlation vs. actual correlation (actual must be \geq to critical).
 - b. Normality diagnostic graph. Normalized residuals must follow and “hug” the 45° line.

Eliminate Violations of the Assumptions

1. Multicollinearity

- a. Deselect high VIF variable (one at a time), or
- b. If two high VIF variables are collinear but both effects are desired in the model then create a new combination variable
 - i. A ratio (income/population)
 - ii. An interaction effect (points scored * games played)

2. Autocorrelation

- a. Try the recommended transformation
 - i. First-difference
 - ii. Durbin-adjusted
- b. If neither solution improves the quality of the regression then report the initial (actual) results and note that an attempt was made to eliminate the violation.

3. Constant Variance

- a. Try weighted-OLS model (WOLS).
- b. If the WOLS solution fails to improve the quality of the regression then report the initial (actual) results and note that an attempt was made to eliminate the violation.

4. Normality

- a. Try maximum likelihood regression (MLE)
- b. Find variables that are non-linear and transform.