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A generalization of modern production theory

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I. INTRODUCTION

The development of duality theory and its application to the economic analysis of production and consumption is a major accomplishment of modern microeconomics. This advance has recently been incorporated into graduate and undergraduate textbooks on microeconomics e.g. Varian (1984) and Russell and Wilkinson (1979), as well as into specialized treatments of consumption and production, such as Deaton and Muellbauer (1980) and Fuss and McFadden (1978). One of the most attractive features of this development is the combination of theoretical generality and elegance of results with the feasibility of empirical application to the analysis of actual productive processes. Notwithstanding, an important characteristic of these productive processes has been consistently neglected in this literature, namely the existence of dramatic variations in capital utilization across firms, industries, countries and over time. That is, there exist substantial variations in the duration of operations of productive processes and these variations are an important part of the phenomena to be explained by a theory of production. For instance, a recently discovered data set by Foss (1981) shows that in the US manufacturing sector the average plant workweek increased by 25% between 1929 and 1976. This increase is due to an increase in the number of hours worked per day through shiftwork.

Curiously enough, in the last twenty years the economic analysis of the duration of operations, usually referred to as capital utilization, employing the standard tools of microeconomics has also developed rapidly. Since the early contributions of Marris (1964) and Georgescu-Roegen (1970), two general surveys have been published (Winston, 1974 and Oi, 1981), and a detailed book length treatment of the topic has become available (Betancourt and Clague, 1981). Moreover, substantial data gathering efforts on the subject have been undertaken by international organizations, e.g. Phan-Thuy *et al.* (1981) and Bautista *et al.* (1981), and national governments, e.g. US Department of Commerce (since 1973). Nevertheless, the powerful tools of duality theory have not been applied to this topic. It is the aim of this essay to begin the process of integrating these two bodies of literature on the economics of production.

As one would expect from a cross fertilization effort, mutual benefits emerge. Thus, the application of duality theory to the analysis of capital utilization leads to substantial generalizations of several of the main results available in the literature as well as to general econometric procedures that can be applied in empirical analyses of this topic. On the other hand, the insights developed in the capital utilization literature when incorporated into the duality theory framework provide a more complete theory of production and lead to procedures

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that eliminate biases which exist in most applications of duality theory to the empirical analysis of production. The end result is a unified framework that can be used to analyse the economics of production with the power and elegance provided by duality theory while incorporating a most important chatacteristic of actual productive processes.

Section II of the paper contains a brief formulation of the firm's long-run capital utilization decision under the usual assumptions employed in the literature. Thus, the firm is viewed as choosing between systems of operations, for example a double-shift system and a single-shift system, when each system is operated at the cost minimizing levels of the decision variables given the economic environment. The formulation in terms of discrete choice facilitates the exposition and econometric analysis; hence, it is widely employed (e.g. Baily, 1976 and Betancourt and Clague, 1981, Ch. 4). In addition it captures more accurately the institutional realities facing any particular firm because wage rates are usually constant within a shift but may vary in stepwise fashion between shifts (Betancourt and Clague, 1981, Ch. 12).

In Section III the economic problem is reformulated in terms of duality theory and the main results available in the capital utilization literature are generalized. A particularly noteworthy feature of this section is that the flexible functional forms associated with duality theory are shown to be quite useful in analysing the effect of technological characteristics of production processes on the utilization decision. The subsequent section contains a general formulation of the cost function which incorporates explicitly the duration of operations into the analysis and reduces to the standard cost function when the duration of operations is exogenously given. In Section V, the econometric implications of the analysis are discussed in the context of estimating the cost function for a cross-section of plants. Constructive results are obtained for this problem. The paper concludes with a brief discussiòn of other implications and suggestions for future research.

Before proceeding to the next section, mention should be made of a related paper. Epstein and Denny (1980) have developed a model where utilization is variable in the short-run, when the capital stock is fixed, but fixed in the long-run, when the capital stock is variable. Duality theory is applied to the analysis of the short-run model in terms of a restricted profit function, which is estimated using aggregate data. As will be seen in the next two sections, the essence of the firm's problem in a long-run framework is a simultaneous choice of the levels of utilization and of the capital stock. Clearly, an integration of a long-run framework with a short-run approach provides a potentially fertile area for future research.

II. THE STANDARD FORMULATION OF THE PROBLEM

The typical analysis of the capital utilization decision considers a firm which wants to produce a stable daily level of output, X, subject to a neoclassical production function, i.e. $X = G(\mathrm{S}, \mathrm{L})$ where L is the 'instantaneous' rate of labour services and S is the 'instantaneous' rate of capital services. The 'instant' is normally defined as an eight-hour shift and $\mathrm{S} = IK$ relates the rate of capital services to the capital stock, K, through a proportionality constant, I, that incorporates the maintenance requirements of the machinery. Since the utilization of equipment over a given calendar period can be varied along two dimensions, duration and intensity, i.e. speed, the assumption that I is constant simplifies the analysis by implying that the speed of operations is constant. Hence, we are focusing on variations in utilization which come about through variations in the duration of operations over a given calendar period. For simplicity of exposition, l is set to unity in the subsequent discussion.

A simple way of describing the firm's problem is as the choice of a system of operations, e.g. a single-shift or a double-shift system, which minimizes the costs of producing the daily level of output subject to the production function constraint. Introducing some notation, the single-shift system will be chosen if

$$rK^1 + WL^1 < rK^2 + W(2+\alpha)L_1^2 \quad (1)$$

where r represents the price of capital or the daily costs of owning a machine, i.e. $r = P_k(i+\delta)$, P_k being the price of a standard machine, i being the daily interest rate and δ the depreciation rate. W is the price of a unit of labour services during normal hours of operation (the first shift), and $\alpha(>0)$ is the shift differential that must be paid to compensate for the inconvenience of work during abnormal hours, i.e. the price of a unit of labour services during the second shift is given by $W(1+\alpha)$. A superscript denotes the system of operations and a subscript denotes the shift of the multiple-shift system, whenever the distinction is necessary.[1] Equation 1 is evaluated at the cost minimizing values of the decision variables for each system and subject, of course, to the production function constraints, which can be written as

$$\bar{X} = X^1 = G(K^1, L^1);\ X^* = 1/2\bar{X} = X_1^2 = G(K^2, L_1^2) \quad (2)$$

This model has been used in the literature on capital utilization to generate several propostions.[2] The first one is somewhat obvious: (a) the higher the shift differential, the higher the costs of the double-shift system and, consequently, the lower the incentive to utilize. Under the assumption of homotheticity, it has been shown that (b) the larger the degree of economies of scale at the process level, the higher the costs of the double-shift system; it has also been shown under this assumption that (c) the high utilization system will always employ a technique, measured by the ratio of capital services to labour services, that is more capital intensive than the one used in the low utilization system. Assuming constant returns to scale, it has been shown that (d) a decrease in the wage rental ratio (W/r) increases (decreases) the incentive to utilize if the elasticity of substitution is less (greater) than unity. With the use of specific functional forms, either Leontief, Cobb–Douglas or CES, it has also been shown that (e) the higher the capital intensity of the technology and (f) the *ex-ante* elasticity of substitution between capital services and labour services, the lower the costs of the high utilization system. These propositions comprise the main implications for capital utilization that have been derived from cost minimization.[3]

[1]In arriving at the expression in Equation 1, two assumptions normally made in the literature have been employed. First, no allowance is made for wear and tear depreciation, which is why the price of capital in both systems remains the same. Second, no allowance is made for possibilities of substitution within the 'day' or calendar period of analysis. The impact of relaxing these assumptions is analysed in, for example, Betancourt and Clague (1981, Ch. 2).
[2]A detailed formulation of these propositions and the relevant references to the literature are available in Betancourt and Clague (1981, Chs 1 and 2).
[3]After this paper was completed, a related unpublished paper by C. Klein was brought to my attention by D. Mueller. In that paper, Klein (1984) establishes four of these six propositions (a, b, d, and e) using the

III. THE DUALITY THEORY APPROACH

In this section the capital utilization decision is formulated terms of the duality theory approach and the above propositions are established or generalized.

In general, duality theory tells us that given a technology with certain characteristics, there exists a cost function, C, with certain properties, e.g. McFadden (1978a). The assumptions in the previous section and the specification of the technology in terms of a production function, $G(S, L)$, derived from a classical production possibilities set in the sense of McFadden, imply that Equation 1 can be rewritten as

$$C^1(W, r, X) = C(W, r, X) < 2C(W^*, r^*, X^*) = C^2(W^*, r^*, X^*), \tag{3}$$

where $W^* = W(2+\alpha)/2$; $r^* = r/2$; $X^* = (\frac{1}{2})X$; and C is a classical cost function for the first shift in each system of operations.[4] C must have the same form for both systems, because it is generated by the same production function. With respect to W and r, C^1 is positive linear homogeneous, concave, continuously differentiable and nondecreasing. With respect to W and r, or W^* and r^*, C^2 has the same properties. Note, however, that C^2 is not homogeneous of degree 1 with respect to W, r and α. $C^1(C^2)$ is increasing and continuous in $X(X^*)$. These properties follow from the definition of a classical cost function (McFadden, 1978b, p. 76). For convenience, the cost function, C, will also be assumed to have these properties: increasing in input prices, differentiable with respect to output and to have second derivatives with respect to input prices.

Establishing the first two propositions is simple. The shift differential affects only the costs of the double-shift system. Therefore, the higher this differential, the higher is W^* and, since $C^2_{W^*} > 0$, the higher are the costs of the double-shift system. When there are economies of scale, a higher degree of returns to scale requires $C_{XX} < 0$.[5] Therefore a higher degree of economies of scale in this situation increases the costs of the double-shift system relative to the single-shift system, because it implies that $C_X < C_{X^*}$ (recall that $X^* = (\frac{1}{2})X$, and $C^1_X = C_X < C^2_X = 2C_{X^*} \cdot \frac{1}{2} = C_{X^*}$). These two factors are the main inhibitors on the cost side to the widespread adoption of shift work.

Footnote 3 continued
duality theory approach and a continuous formulation of the choice of duration. Klein's paper does not address, however, the other two propositions or the implications of these results developed in subsequent sections.

[4]The left-hand side of Equation 3 follows from the definition of the cost function in the standard problem; the right-hand side of Equation 3 follows from applying the same definition to the cost of the *first* shift of the double-shift system. That is, notice that the right-hand side of Equation 1 can be rewritten as

$$rK^2 + W(2+\alpha)L_1^2 = 2\{(r/2)K^2 + [W(2+\alpha)/2]L_1^2\} = 2\{r^*K^2 + W^*L_1^2\},$$

where the expression in curly brackets represents the costs of the first-shift of the double-shift system.

[5]The degree of economies of scale is inversely related to $E_{CX}(= C_X X/C)$. An increase in the degree of returns to scale thus implies

$$C_{XX}(X/C) + (C_X/C) - C_X X C_X/(C)^2 = C_{XX}(X/C) + (C_X/C)(1 - E_{CX}) < 0.$$

If there are economies of scale, $E_{CX} < 1$; and $C_{XX} < 0$ is a necessary condition for a higher degree of economies of scale in this region.

In order to establish the next two propositions, it will be assumed that the production function is homothetic. It immediately follows (Uzawa, 1964) from this assumption that Equation 3 can be rewritten as

$$C^1(W, r, X) = C(W, r)k(X) < C^2(W^*, r^*, X^*) = 2C(W^*, r^*)k(X^*). \tag{4}$$

One can now show that the higher utilization system will always operate with a technique that is more capital intensive than that of the low utilization system. From the derivative property (Shephard's Lemma, e.g. Varian, 1984, p. 54), it follows that

$$K^1/L^1 = C_r(W, r)/C_W(W, r) \tag{5}$$

and

$$K^2/L_1^2 = C_{r^*}(W^*, r^*)/C_{W^*}(W^*, r^*). \tag{6}$$

Since $W^* > W$ and $r^* < r$, $C_{W^*} < C_W$. The concavity of the cost function implies that $C_{W^*W^*} < 0$ and, together with linear homogeneity,[6] $C_{W^*r^*} > 0$. By a similar argument, it follows that $C_{r^*} > C_r$. Therefore, on both accounts $K^2/L_1^2 > K^1/L^1$.[7] Incidentally, this propostion provides one reason for the often cited positive association between observed capital intensity and observed utilization levels (e.g. Betancourt and Clague, 1981, Ch. 5). Another important reason is the role of capital intensity as a determinant of capital utilization, which is analysed below in connection with Equation 11. The implication of both results together is the existence of a simultaneity problem in the empirical analysis of capital utilization, which is one of the issues analysed in Section V.

To show that an increase in the wage rental ratio decreases (increases) the incentive to utilize if the elasticity of substitution is less (greater) than unity, two steps are necessary. First, notice that the sign of the effect of an increase in the price of capital, for example, on the ratio of the costs of system 2 to the costs of system 1 is given by

$$\partial(C^2/C^1)/\partial r = [C_{r^*}k(X^*)C^1 - C_r k(X)C^2]/(C^1)^2 \gtreqless 0,$$

or, after some manipulation,

$$\partial(C^2/C^1)/\partial r = (\theta_{1K}C^2/C^1)[\theta_{2K}/\theta_{1K} - 1] \gtreqless 0, \tag{7}$$

where θ_{iK} is the share of capital costs in the combined labour and capital costs of the ith system of operations. Therefore, an increase in the price of capital or a decrease in the wage rental ratio increases (decreases) the incentive to utilize if $\theta_{2K} < \theta_{1K}(\theta_{2K} > \theta_{1K})$.

Second, a move from system 2 to system 1 can be viewed as a decrease in the wage rental ratio or an increase in the price of capital, and what happens to the capital share in these circumstances will be given by

$$\partial(rC_r/C)/\partial r = [C_r C + rC_{rr}C - r(C_r)^2]/(C)^2,$$

[6]That is, the linear homogeneity of $C(r^*, W^*)$ implies $-C_{r^*W^*} = W^*/r^* C_{W^*W^*}$, but by concavity $C_{W^*W^*} < 0$.

[7]Note that it also follows from Equation 6 that the higher the value of α, the higher the capital intensity of the production process.

or, after some manipulation,

$$\partial(rC_r/C)/\partial r = (W/(C)^2)[1 - C_{rw}C/C_rC_w]C_rC_w = (WC_rC_w/(C)^2)[1-\sigma] \tag{8}$$

The second equality in Equation 8 follows from the definition of the elasticity of substitution in terms of the characteristics of the cost function (McFadden, 1978b, p. 79). Therefore if the elasticity of substitution is less (greater) than unity, and remains so over the range $[(W^*, r^*), (W, r)]$, an increase in the price of capital or equivalently a decrease in the wage rental ratio, increases (decreases) the incentive to utilize, because under these circumstances $\theta_{2K} < \theta_{1K} (\theta_{2K} > \theta_{1K})$. As Winston (1974) has noted, this proposition is important because it provides the basis for the analysis of many policy issues that affect capital utilization.

The last two propositions identify technical characteristics of productive processes which have a positive influence on the incentive to utilize, i.e. the capital intensity of the technology and the range of *ex-ante* substitution possibilities available in the technology. Consequently, the results present in the capital utilization literature have been obtained under the assumption of specific functional forms for the production function, specifically Leontief, Cobb–Douglas or CES. The procedure employed here is to demonstrate these results for the generalized Leontief cost function (Diewert, 1971). This functional form is a parsimonious flexible form, i.e. it is an accurate second order approximation to an arbitrary classical cost function in a small neighbourhood of a point (Fuss *et al.*, 1978). Since the characteristics of the technology that we are examining (capital intensity and substitution possibilities) involve only first and second derivatives of the cost function, a second order approximation suffices for our purposes. The generalized Leontief form was selected from among other parsimonious flexible forms because of simplicity in establishing the results.

Consider the specification of the cost function in terms of a generalized Leontief for both systems of operation,

$$C^1 = C(r, W)k(X) = [b_{11}W + 2b_{12}(Wr)^{1/2} + b_{22}r]k(X) \tag{9}$$

$$C^2 = 2C(r^*, W^*)k(X^*) = 2[b_{11}W^* + 2b_{12}(W^*r^*)^{1/2} + b_{22}r^*]k(X^*), \tag{10}$$

where $b_{12} > 0$. The parameters of the cost function in Equations 9 and 10 are related to the technical characteristics of the technology. Specifically, the conditional factor demand functions are such that the higher the value of the b_{ii} parameter, the higher is the demand for the ith factor of production for any given level of factor prices; hence, given two technologies (A and B), if $b_{22}^A > b_{22}^B$, other things equal, technology A is more capital intensive than technology B. Also, as Varian (1984, p. 181) indicates, the higher the value of $b_{ij} (i \neq j)$ the higher the elasticity of substitution. With these characterizations of the implications for the technology of the generalized Leontief cost function, one can turn to establishing the two propositions of interest.

The first conceptual experiment to be performed is the following: if two firms face the same factor prices and output levels but one of them (A) has a more capital intensive technology than the other (B), the firm with the more capital intensive technology will have relatively lower costs than the other in the operation of the double-shift system. To wit, $(C^2)^B - (C^1)^B > (C^2)^A - (C^1)^A$, which implies $(C^1)^A - (C^1)^B > (C^2)^A - (C^2)^B$, or, using Equations 9 and 10,

$$k(X)[b_{22}^A - b_{22}^B]r > 2k(X^*)[b_{22}^A - b_{22}^B]r^*. \tag{11}$$

Since $r^* = r/2$ and $k(X^*) = k[(1/2)X]$, which is always smaller than $k(X)$ because the cost function is increasing in output, Equation 11 always holds and high levels of capital intensity make more attractive the high utilization system.

The second conceptual experiment to be performed follows. Consider two firms that face the same factor prices and output levels but one of them (A) has a technology with a higher elasticity of substitution than the other (B). The firm with the higher elasticity of substitution technology will have relatively lower costs than the other in the operation of the double-shift system. That is, $(C^2)^B - (C^1)^B > (C^2)^A - (C^1)^A$, which implies $(C^1)^A - (C^1)^B > (C^2)^A - (C^2)^B$, or, using Equations 9 and 10,

$$k(X)2W^{1/2}r^{1/2}[b_{12}^A - b_{12}^B] > 2k(X^*)2(W^*)^{1/2}(r^*)^{1/2}[b_{12}^A - b_{12}^B] \tag{12}$$

Since $r^* = r/2$ and $W^* = W(2+\alpha)/2$, Equation 12 can be rewritten as

$$k(X)/k[(1/2)X] > (2+\alpha)^{1/2}. \tag{12'}$$

If there are constant returns to scale, $k(X) = X$ and $k[(1/2)X] = (1/2)X$; therefore, Equation 12′ becomes $2 > (2+\alpha)^{1/2}$ and the proposition is valid for $0 < \alpha < 2$. Since observed shift differentials range from 0 to 0.45 (Betancourt and Clague 1981, pp. 224–7, and the references cited therein), one can confidently assert that Equation 12′ holds under constant returns to scale. If there are increasing returns to scale, however, $k(X)/k[(1/2)X]$ moves toward unity (from the constant returns to scale value of 2) as the degree of economies of scale increases and Equation 12′ will not hold for values of α above some critical value α^* such that $0 < \alpha^* < \alpha < 2$. Hence, for a classical cost function the propostion is valid as a general tendency rather than as a categorical statement.

To conclude this section, it is useful to put in perspective the last two results. First, the proposition established here, that high levels of capital intensity in the technology lead to high levels of utilization, is considerably stronger than what exists in the capital utilization literature, because it holds for any homothetic technology consistent with an arbitrary classical cost function and not just for the CES technology or its special cases. The same characterization applies to the relation between utilization and the elasticity of substitution, provided one assumes constant (or decreasing) returns to scale. If there are increasing returns to scale, however, the result obtained in the utilization literature for the CES functional form need not apply to the more general production technologies allowed by a classical cost function.

IV. A GENERAL FORMULATION OF THE COST FUNCTION

In the previous two sections attention has been centred on the choice between a single-shift and a double-shift system of operations to bring out the essential economic features of the choice of duration of operations. The generalization of these results, however, is easily accomplished. That is, in general the cost function will be given by

$$C^* = d \cdot C(W^*, r^*, X^*) \tag{13}$$

where $W^* = W[d+\alpha(d)]/d, r^* = r(d)/d, X^* = X/d$ and d is an index representing the duration of operations.[8] The function C has the properties indicated in the previous section.

In relating our results to the modern theory of production, it is convenient to adopt the convention that the index d takes on the value of unity for a normal or reference period of duration of operations, e.g. an eight-hour shift, and that the α function has the property $\alpha(1) = 0$. With this convention, the formulation of the cost function in Equation 13 generates the standard formulation in the literature when $d = 1$. In order to highlight the importance of this result, it is useful to note that Varian (1984, p. 37), for example, puts forth the standard cost function as the single most powerful tool for studying the economic behaviour of the firm, because it summarizes all economically relevant information about the technology of the firm. Since the duration of operations is an important economic variable, the characterization of the cost function given by Varian applies with greater force to the formulation in Equation 13 than to the standard formulation. Indeed, the cost function in Equation 13 represents the solution to a two-stage optimization procedure. That is, for every possible system of duration of operations, d, the decision maker chooses the values of the other variables that minimize costs given the system; subsequently, the system of duration of operations that entails the lowest possible cost is selected. While the specification of the cost function in Equation 13 incorporates all the economically relevant information on this second choice, the standard formulation ignores this choice.

For future reference, it will be useful to discuss in detail the nature of the index, d, and of the function, $\alpha(d)$. The index d represents a ratio scale variable, which may or may not be an integer.[9] Thus, there is meaning to the statement that a three eight-hour shift system lasts three times as long as a single eight-hour shift system. The actual duration premium, or the segment of the α function on which a particular firm operates, normally depends on the choice of system; but the determinants of the α function, other than duration, are exogenous to the firm. For instance, in the case of an environment where the firm faces a shift differential for the evening shift (α) and another (higher) one for the night shift (α') the α function can be defined as follows: if $0 < d \leqslant 1$, $\alpha(d) = 0$; if $1 < d \leqslant 2$, $\alpha(d) = \alpha(d-1)$; and if $2 < d$, $\alpha(d) = \alpha + \alpha'(d-2)$.

The shift differentials are exogenous, but the segment of the α function where the firm operates is chosen by the firm through its choice of duration. An interesting property of the model is that one does not need to know these differentials or other characteristics of the system

[8] Henceforth, we are allowing the cost of the capital stock to depend also on the choice of duration of operations, i.e. $r(d)$, in order to allow for wear and tear depreciation due to additional usage. An explicit discussion of the effects of this more general formulation and a sensitivity analysis of its impact on the price of capital *services*, r^*, is available in Betancourt and Clague (1981, Ch. 2, Section 2). Nonetheless, the main implication of this analysis can be summarized as follows: the increase in the cost of the capital *stock* due to additional wear and tear through a lengthening of duration is not sufficient to lead to an increase in the price of capital services. That is, while $r(d)$ rises as d rises, $r^*(= r(d)/d)$ still falls as d rises. This result suffices to ensure the applicability of the subsequent analysis.

[9] Given the assumptions of the theory presented in Section III, the firm would never choose partial shifts and d would only take on integer values. Nevertheless when *ex-post* realizations differ from *ex-ante* expectations, partial shifts would be observed and d need not be an integer. Of course, extensions of the theory also lead to noninteger values for d. For instance, the existence of seasonal output fluctuations would normally lead to partial shifts, Betancourt and Clague (1981, Ch. 3, Section 4), and to noninteger values for d.

of duration of operations in order to calculate the effective wage rate faced by the firm. To wit, $W^* = W[d+\alpha(d)]/d$, but W^* can be measured by calculating the ratio of the total wage bill, $W(d+\alpha(d)]L(d)$, to the total number of man hours $dL(d)$. Since data on shift differentials are not easily available, this feature of the model can be useful in practical applications.

V. ECONOMETRIC IMPLICATIONS

The econometric implications of the cost function in Equation 13 are far reaching. In general it suggests that applications of modern production theory which ignore the duration of operations, by implicitly assuming that $d = 1$, are usually subject to significant measurement error in the independent variables, to simultaneity biases, or both. It is well known that under these circumstances econometric results are of dubious value. To understand the reason for the conclusion and to provide the basis for subsequent discussion, it is convenient to consider three situations, starting with the simplest and proceeding to the more complex. Furthermore, for the sake of clarity, the discussion in this section will be developed in the framework of a sample of observations from a cross-section of plants.

Suppose that the duration of operations is exogenously given at the same value for every plant in the sample. By choosing this actual value of duration to be the standard or reference point at which $d = 1$, the cost function in Equation 13 collapses to the standard one and the researcher can proceed as before. More interestingly, suppose instead that the duration of operations is exogenously given at a different value for every plant in the sample. If the empirical researcher has information on the actual duration of operations, all he needs to do is calculate $r(d)$ and deflate $r(d)$ and X by the value of d, recall that the standard way of calculating the wage rate empirically yields W^*, and the estimation of Equation 13 can take place with exactly the same methods as before using (d, r^*, W^*, X^*). Unfortunately the theory of capital utilization, of which the principal features are summarized in Section III, teaches us that the most relevant situation is one where duration is an endogenous variable determined by economic considerations. Therefore, the application of the deflation procedure outlined for the previous case and the use of d will in general introduce a simultaneity bias in the application of standard methods of estimating the cost function. Moreover, if one fails to use d and the deflation procedure, as Equation 13 shows, there will be substantial and systematic measurement error in the independent variables.

How should econometric analysis proceed in the face of this difficulty? To some extent, it will depend on the specific purpose of the analysis. To keep the discussion brief, however, the focus will be on the general case where the analysis requires estimation of the cost function or of all its parameters.[10]

There are two general solutions to the simultaneity problem introduced by endogenous capital utilization in the context of estimating the cost function. The first solution lies in the

[10] If the purpose of the analysis requires only the estimation of substitution possibilities, the simultaneity problem can be dealt with through a proper definition of the variables (Betancourt, 1984). Nevertheless, if the analysis requires the estimation of parameters associated with economies of scale or technological change the econometric problems discussed here have to be addressed directly.

stratification of the sample of plants by the systems of duration of operations; the second solution lies in obtaining instrumental variables for all the variables that are functions of the duration of operations. In either case, however, no new estimation methods are required with respect to the estimation of the cost function itself. Therefore, we will briefly point out the main avenues available in the literature for estimating the cost function.

As Varian (1984, Ch. 4) points out, for example, the most typical way of estimating the parameters of the cost function is to postulate a flexible functional form for the cost function and apply Shephard's Lemma to this functional form to obtain the conditional input demand functions. For instance, under constant returns to scale, the generalized Leontief cost function yields a system of linear conditional input demand functions and the translog cost function yields a system of linear share equations for the inputs. Applications of standard estimation methods to these linear systems yield estimates for the parameters of the cost function. Under more general specifications of economies of scale, the same procedure still yields a system of conditional input demand functions or share equations but system estimation methods that include the cost function would be usually required, e.g. Christensen and Greene (1976).

Incorporating the existence of endogenous variations in the duration of operations into these methods can be accomplished in a most straightforward manner. Let us suppose first that one can identify in the sample of plants a small number of different systems of duration of operations, e.g. a single eight-hour shift, a double-shift system of two eight-hour shifts and a triple-shift system of three eight-hour shifts. In these circumstances, the simultaneity problem can be taken care of by simply stratifying the plants into mutually exclusive categories corresponding to each of the systems. Whatever estimation methods are viewed as having desirable statistical properties in the absence of variations in the duration of operations will have equally desirable properties when applied to each group separately, provided that the calculated price of capital and the level of output are properly deflated for the nonreference groups. That is, for the two nonreference groups in our previous example, let us say the double-shift system and the triple-shift system, one needs to use ($r^* = r(2)/2, X^* = X/2$) and ($r^* = r(3)/3, X^* = X/3$), respectively.[11]

Alternatively, let us suppose that there are enough duration systems to mean that stratifying the sample is not a feasible alternative. For example some groups may contain very few observations. In these circumstances, one needs to obtain appropriate instrumental variables which can be used to replace d, r^*, W^* and X^* in Equation 13 with $\hat{d}, \hat{r}^*, \hat{W}^*$ and $\hat{X}^*$. With these substitutions, the estimation of Equation 13 can proceed using exactly the same methods as before, although the statistical properties of the estimates will be weakened due to the reliance on instrumental variables. A general procedure for obtaining appropriate instruments is suggested below; here, it suffices to note that these instruments will be the predicted values of reduced form equations. Therefore, when the parameters of the cost function are estimated on the basis of a least squares procedure, the properties of the estimators would be those of nonlinear two-stage least squares (Amemiya, 1974).

Implementation of this second alternative requires the specification of reduced form equations for the duration of operations and the other endogenous variables. Given the general

[11]Recall from the previous section that the use of the average wage for the calendar period of analysis automatically yields W^*.

form of the cost function in Equation 13, an appropriate general form for a reduced form equation would be, for example,

$$d = f(W/r(1), X, Z) + U \tag{14}$$

where Z represents the exogenous characteristics of the systems of duration of operations and U is a stochastic error term. Similar reduced forms would be needed for r^*, W^* and X^*.

Economic theory tells us that the factor prices relevant for the reference system of operations, i.e. $d = 1$, enter into Equation 14 as a ratio because the cost function is linear homogeneous in r and W for all possible duration systems. Moreover, econometric theory and practice also provides some guidance on the specification of the reduced form equations. More precisely, if the functional forms of the reduced form equations are not known, as is the case here, they can be approximated by polynomials. If the polynomials are of the same degree in all the reduced form equations, the two-stage least squares estimators would be consistent (see Kelejian, 1971).[12]

Finally, a number of practical issues should be noted. First, the relevant factor price ratio in Equation 14 is $W/r(1)$ not W^*/r^*. Therefore, for those firms operating with a duration system other than the standard or reference one, measurement of this variable requires information on the exogenous characteristics of the systems of duration of operations. Second, the relevant level of output in Equation 14, X, is what the ith plant would have produced under the standard or reference system of duration of operations, but in this case the information is readily available in the data given the existence of information on duration. Last but not least, the information on the exogenous characteristics of the systems of duration of operations in Equation 14, e.g. the evening and night shift differentials, is needed for all plants in the sample, including the ones using the standard or reference system. These information requirements are difficult but not impossible to meet.

To summarize, in estimating the cost function with cross-section data, the identification of the basic systems of duration of operations is a very rewarding effort for two reasons. First, the information requirements of the estimation procedure that relies on stratification by the system of duration of operations are less stringent. Second, the resulting estimators will have more desirable statistical properties than under the alternative procedure, which requires the estimation of Equation 14. Hence, it is worth noting that, in their extensive study involving over 2000 British establishments, the National Board for Prices and Income (1970) identified eight 'basic' shift systems. Moreover some of these basic systems can be treated as the same for our purposes, because they involve the same duration of operations, e.g. double-days and days and nights.

VI. CONCLUDING REMARKS

A very simple but important implication of the analysis in this paper applies to the work of statistical collection agencies that gather information on establishments. The empirical analysis

[12]One needs to obtain instruments for $r^* = r(d)/d$, $X^* = X/d$ and $W^* = W(d + \alpha(d))/d$ from the reduced form equations. The simpler procedure of estimating $\hat{d}$ from Equation 14 and using this estimate to obtain $\hat{r}^* = r(\hat{d})/d$, for example, would lead to inconsistent estimates in the second stage, because of the nonlinearity.

of production is seriously incomplete in the absence of information on the duration of operations; furthermore, information on the characteristics of different systems of duration of operations, e.g. shift differentials, is also highly desirable for this purpose. For instance even the unique census surveys of plant capacity (US Department of Commerce) mentioned earlier, which contain very useful information on the duration of operations, lack information on shift differentials. In this connection, it should perhaps be mentioned that information on capacity utilization, e.g. output-based percentages, is frequently of no use as a substitute for information on capital utilization or the duration of operations. Indeed, the typical adjustments made to empirical production data using capacity utilization measures are designed to account for departures between *ex-ante* designs and *ex-post* realizations and they would also be applicable regardless of the system of duration of operations.

The general formulation of the cost function developed here can easily be extended to more than two inputs, provided due attention is paid to the definition of the prices of the additional inputs. It is certainly straightforward to incorporate the duration of operations into the analysis of productive processes with, for example, various types of capital and labour. Nonetheless, if the other inputs involve fuel or materials, the situation becomes complicated by the nature of continuous process technologies. For these technologies there are significant cost savings as a result of employing high utilization systems and, consequently, the relevant fuel or materials price to use in the cost function is a function of utilization, e.g. Betancourt and Edwards (1984).

An obvious area for future research is the econometric analysis of capital utilization employing the duality theory approach. The formulation of the cost function in Equation 13 provides the basis for empirical analysis of the choice of duration. That is, once the parameters of the cost function have been estimated by the methods discussed in the previous section, it becomes possible to explain empirically the choice of system of duration of operations by, for example, comparing the costs of any two systems of duration of operations. The costs of the two systems can be calculated using the estimated parameters of the cost function and the information on the exogenous variables. In other words, the experiment suggested here is simply an empirical explanation of the solution to the second stage of the optimization procedure embedded in the formulation of the cost function in Section IV. The development of estimation methods for explaining this second stage decision is still in its infancy.

More generally, the framework developed in this paper provides a useful starting point for the theoretical and empirical analysis of a wide range of issues in the economics of production. Whatever the specific objective of production analysis, e.g. scale, distribution, substitutability, separability, technical progress, index numbers or capital utilization, the specification of the cost function developed here allows the powerful and elegant tools of duality theory to be applied to a problem while incorporating the pervasive role of the duration of operations into the analysis.

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