R. W. Bethlehem, BCom (Witwatersrand)

Defining the cost of capital

INTRODUCTION

It is generally recognised that profit is arrived at by deducting costs from revenues. The calculation of profit, however, is often a complex matter because many problems arise in the estimation of these two basic variables. Costs, in particular, are a source of difficulty because it is necessary to distinguish not only between those of an operating and financial nature but also between out-of-pocket and opportunity costs. The latter are no less real even though they have no place in conventional accounting systems. If it is accepted that the employment of any resource or production factor can only be justified if its contribution to total revenues exceeds the addition that it necessitates to total costs then it is necessary, too, to take account of opportunity costs. This is especially so in the case of the financial liabilities of an enterprise of which three basic categories, loan, preference and equity, must be distinguished. Each of these involves the issue of different classes of securities to investors which, because of differences in risk and growth, result, also, in differences in cost or yield.

WHY IT IS IMPORTANT TO CALCULATE THE COST OF CAPITAL

The importance of calculating the cost of capital stems from the fact that management, if it is to maximise profits or present net wealth, must have a basis for evaluating new and sometimes competitive investment projects. The cost of capital, thus, becomes the minimum return which must be earned if the position of present shareholders is not to be undermined. In other words, unless a new project adds more to income flows than it adds to outlays, allowing for the timing of both, new funds should not be raised to finance it. To minimise the average cost of capital, however, it is necessary to properly appraise the relative costs of alternative capital categories and it is to this that we now must turn.

DIFFERENTIATING BETWEEN DIFFERENT TYPES OF CAPITAL COST

1
The cost of loan capital

The cost of loan capital may, *as a first approximation*, be defined as the rate of return that must be earned by loan financed investments in order to keep unchanged the earnings available to ordinary shareholders.[1] Thus, if the rate of interest charged on loan funds is 8% p.a., this will be seen to be the cost of such capital. Account, however, has to be taken of three additional factors before the final cost of loan capital can be determined, namely:

(a) the deductibility of loan interest as an expense in the determination of a firm's taxable income;

(b) the adverse effect of increases in loan capital on capital gearing and hence equity risk and value; and

(c) the flotation costs of new loan issues.

Because loan interest is a tax deductible expense, the real cost of debt, *ceteris paribus*, will always be less than its contractual rate. In effect, the tax authorities subsidise loan finance. Thus, at a 50% tax rate, an 8% loan would only cost 4% in fact, i.e. the cost of loan capital (Ki) would be:

$$\frac{i\,(1\ -\ q)}{Pl}$$

where Pl = bond price, i = interest rate and q = corporate tax rate.

Against this have to be set the losses that arise from the fall in the value of equity securities when gearing, and hence risk, is increased, and the proportionately limited but, nevertheless, significant flotation expenses such as legal fees, underwriting commission, stock exchange charges, printing and stationery etc., incurred with new loan issues. Allowing for these, Ki becomes:

$$\frac{i\,(1\ -\ q)}{Pl\ -\ f}\ +\ u$$

where f = flotation costs and u = imputed costs associated with the deterioration of equity values as a result of a higher loan-equity ratio. The deterioration of equity values need have nothing to do with the actual level of equity earning per share. Even were these to be increased because of the investment of the new funds, the value of the equity might decline because the earnings have become more risky. To, therefore, assess whether the new project should be undertaken or not, it is necessary to establish whether the quantitative gain (as measured by the increase in earnings per share) is greater or less than the qualitative loss (as measured by the increase in the dividend or earnings capitalisation factor). If the former, the project should be undertaken; if the latter, it should not be. The loss in equity value resulting from greater use of loan capital is real to shareholders, even though it is an 'imputed' rather than a direct or out-of-pocket cost.

From the above an immediate source of difficulty becomes apparent. Though the imputed cost is vital to the calculation, it cannot be determined precisely until the new loan capital is already raised. On the other hand, the cost of loan capital must be known before a loan issue is made if a proper evaluation of investment projects is to be made. The only way out of this dilemma, is to rely on estimates of the imputed cost

1 J. Fred Weston & Eugene F. Brigham, *Managerial Finance*, 3rd Ed., 1969, p. 341

even though such estimates may prove to be inaccurate because the stock market is unsettled.

These factors considered, the definition of the cost of loan capital, given above, can be modified to replace the phrase, 'to keep unchanged the earnings available to ordinary shareholders', with the phrase, 'to keep unchanged the wealth of ordinary shareholders'.

2
The cost of preference capital

Because of the fixed income nature of preference shares, the cost of preference capital can be defined in a similar manner to the cost of loan capital. It is important, however, to take account of the following points of difference:

(a) unlike loan capital, failure to meet the fixed commitments of preference capital does not confront an enterprise with a threat of bankruptcy;
(b) the income paid on preference capital is not deductible as an expense for tax purposes;
(c) preference share dividends in the hands of investors are subject to a measure of tax exemption; and
(d) though preference capital has a gearing effect similar to loan capital when company profits are growing, its reverse gearing effect is limited by its proprietor status and it does not, therefore, have an imputed cost similar to that of loan capital.

3
The cost of equity capital

Risk and flotation costs aside, the position of ordinary shareholders will be improved by any new investment which will add something positive to earnings per share. The cost of external equity capital (Ke), therefore, can be roughly defined as earnings per share (Ee) divided by market price (Pe). This will be seen to be the equivalent of the current earnings yield as determined by market forces at any time. Thus, a rise in price and or fall in earnings yield will result in a fall in a company's cost of equity capital, and vice versa.

This definition, however, only applies if Ee is presumed to be the same in future years as it is at present. If a growth of future earnings is assumed, and *ceteris paribus*, growth would result if part of current earnings, instead of being paid out to shareholders, is ploughed back for investment within a firm, the definition requires modification.

Before dealing with this modification, it will serve some purpose to restate the non-growth definition of the cost of new equity capital, changes in risk or gearing and flotation costs ignored, as follows:

$\frac{Ee}{Pe} = \frac{De}{Pe}$ where De = equity dividends.

This will be so, because in a non-growth situation Ee = De by definition.

When introducing growth to the definition of the cost of equity capital, it is immediately clear that a distinction has to be made between 'expansion growth' (gx), i.e. that growth resulting merely from retentions, basic profitability remaining unchanged, and that growth resulting from actual improvements in profitability and efficiency (ga). Total growth (gt) might be made up of both these forms of growth. Thus gt = gx + ga.

It can be shown that the value of an ordinary share is given by the following equation:

$$Pe = \frac{De}{Ke - gt}$$ [2] (see footnote)

where Pe = price or value; De = dividend per share; gt = total growth factor and Ke = cost of equity capital.

If this is accepted, the latter can be defined thus:

$$Ke = \frac{De}{Pe} + gt$$

But

$$\frac{De}{Pe} = \frac{Ee}{Pe} - gx \text{ if } De < Ee$$

where gx is defined as $\frac{Ee\left(\frac{Ee - De}{Ee}\right)}{Pe}$ and $\frac{Ee - De}{Ee}$ is the ratio of earnings retained,

i.e. $\frac{De}{Pe} = \frac{Ee}{Pe} - \frac{Ee\left(\frac{Ee - De}{Ee}\right)}{Pe}$

$= \frac{Ee}{Pe} - \frac{(Ee - De)}{Pe} = \frac{De}{Pe}$

Therefore,

$$Ke = \frac{Ee}{Pe} - gx + gt = \frac{Ee}{Pe} + ga$$

In other words, provided all other things remain unchanged, it does not really matter whether the cost of

---

[2] **Proved as follows:**

Let, growth factor = gt; next dividend = De
discount factor = Ke; present value of div. stream = Pe

Then,

$$Pe = \frac{De}{Ke} + \frac{De.gt}{Ke^2} + \frac{De.gt^2}{Ke^3} \cdots\cdots \frac{De.gt^{n-1}}{Ke^n}$$ **Stage I**

$$= De\left\{\frac{1}{Ke} + \frac{gt}{Ke^2} + \frac{gt}{Ke^3} \cdots\cdots \frac{gt^{n-1}}{Ke^n}\right\} \times \frac{gt}{gt}$$ **Stage II**

$$= \frac{De}{gt}\left\{\frac{gt}{Ke} + \frac{gt^2}{Ke^2} + \frac{gt^3}{Ke^3} \cdots\cdots \frac{gt^n}{Ke^n}\right\}$$ **Stage III**

The expression within the brackets is a geometric series the summation of which is given by the formula

$$S = \frac{a(1 - r^n)}{1 - r}$$

where, n = number of terms; a = first term; r = common ratio and S = total sum.

Therefore,

$$Pe = \frac{De}{gt}\left\{\frac{\frac{gt}{Ke}\left(1 - \frac{gt^n}{Ke^n}\right)}{1 - \frac{gt}{Ke}}\right\}$$ **Stage IV**

$$= \frac{De}{gt}\left\{\frac{\frac{gt}{Ke} - \frac{gt^{n+1}}{Ke^{n+1}}}{1 - \frac{gt}{Ke}}\right\}$$

As long as $gt < Ke$ and $n = \infty$, $\frac{gt^{n+1}}{Ke^{n+1}}$ must tend to zero.

Thus,

$$Pe = \frac{De}{gt}\left\{\frac{\frac{gt}{Ke}}{1 - \frac{gt}{Ke}}\right\} = \frac{\frac{De}{Ke}}{1 - \frac{gt}{Ke}}$$ **Stage V**

However,

$$\frac{1}{1 - \frac{gt}{Ke}} = \frac{Ke}{Ke - gt}$$ **Stage VI**

Therefore,

$$Pe = \frac{De}{Ke} \times \frac{Ke}{Ke - gt} = \frac{De}{Ke - gt}$$ **Stage VII**

equity capital is stated in terms of dividends or earnings. The two amount to exactly the same thing. For the sake of convenience, in what follows, we will use the definition

$$Ke = \frac{De}{Pe} + gt$$

It can, thus, be seen that the cost of equity capital, like the cost of loan and preference capital, is essentially an opportunity cost. That is, the cost of equity capital is that rate of return that must be earned on new equity funds if the wealth of ordinary shareholders is not to be impaired in any way.[3]

THE IMPACT OF FLOTATION COSTS, IMPUTED CHARGES AND SHAREHOLDERS' TAX ON THE COST OF EQUITY CAPITAL

Thus far, in considering the cost of new equity capital, three crucial factors have been left out of account, namely:

(a) the flotation costs of new equity issues;
(These reduce the actual level of P and, thus, raise Ke.)

(b) the reverse effect of imputed charges when new equity capital is issued by a company whose capital structure already contains debt;
(This by lowering the loan-equity ratio and hence risk, will tend to lower Ke through a beneficial effect on P.)
and

(c) the tax on dividends in the hands of ordinary shareholders.
(This makes the financing of new corporate investments through new equity issues less advantageous than their financing through retentions because the amount made available to shareholders and which could be used by them for subscribing to new shares, is lowered to the extent of the tax.)

The last point indicates that the opportunity cost of external funds is greater than the opportunity cost of internal funds. The company concerned, however, would have to take note also of the effect on its share price of its cutting its current rate of dividend or of its failing to pay as high a rate as would have been possible had the investment project concerned not been undertaken. Although, theoretically, cutting the dividend should make no difference because investors are assumed to be rational and aware of the importance of earnings, the effect of dividend cuts or restraints cannot be ignored in the real world.

GROWTH AND THE COST OF EQUITY CAPITAL

It is clear from the above, that in a world of change, growth is a vital factor in the estimation of a company's cost of capital. There is nothing certain about growth, however. Though it may be expected to be great, it might turn out to be negligible or even negative. It cannot be known beforehand with any exactitude. This means that in calculating a company's cost of equity capital one is forced to depend on a subjective variable, for if growth cannot be known precisely it must be estimated. But the estimates of different parties may not agree, in which case there will be an absence of agreement as to a company's cost of capital. Differences in estimates of capital costs may be poignantly illustrated by a divergence between market prices and estimates by management as to a particular company's intrinsic worth.

THE COST OF CAPITAL AND THE RATE OF INTEREST

Given such disagreement, which view should prevail? There is no way of satisfactorily answering this question. Solomon, however, notes that the way K is derived must depend on the conditions on which the enterprise is able to obtain funds in the capital market.[4]

In a world of complete certainty K would simply equal i, the rate of interest, i.e. the cost which relates money to user time preferences. This would be so because in such a world, the expected rate of return on new investments would be as certain and as definite as the cost of capital. In other words, in a world of complete certainty, there would be no distinction between equity and loan funds and hence between different kinds of securities.[5]

In a world of uncertainty, however, differences do exist between loan stocks and equities and the rate of interest, too, is subject to change. This means that true profitability can have no absolute expression. An investment that is considered to be worthwhile at one point in time may not be at another because the cost of capital has changed.

DEFINING THE COST OF EQUITY CAPITAL

The above considered, the cost of equity capital can be defined as the minimum rate of return that must be earned on equity financed investments to keep unchanged the value of the existing ordinary shares of a company.[6]

The definition applies in particular to retained earnings. With new equity capital, account has to be taken of the extra cost associated with flotation charges and the taxation of dividends in the hands of shareholders. This said, the equations relating to the cost of equity capital can be restated as follows:

(a) Cost of retained earnings (Ke_1)

$$Ke_1 = \frac{De}{Pe} + gt$$

(b) Cost of new equity capital (Ke_2)

$$Ke_2 = \frac{De}{\frac{(1 - x)}{Pe - f}} + gt$$

3 In economic theory, opportunity cost is the sacrifice of the alternative foregone in producing a good or service. Relating this notion to the cost of capital, it is clear that in any particular capital budgeting situation, management, given the means of finance, has two choices. Either it can vote for undertaking the project in question, or it can vote to leave the business situation unchanged. As an unchanged business situation implies a certain return on equity, it follows that a decision to go ahead necessitates that that return at least be earned if the position of ordinary shareholders is not to be undermined. It is in this sense that the cost of capital is described as an opportunity cost, i.e. the return on the alternative foregone.

4 Ezra Solomon, *The Theory of Financial Management,* Columbia Univ. Press., 1967, p. 28

5 Ibid. p. 29

6 Weston & Brigham, op. cit. p. 344

where x = shareholders' tax rate; and f = flotation costs per share.

From these equations, it is clear that the cost of equity capital is dependent, in any dynamic situation, on three basic variables, namely D, P and g. None of these can be known with absolute certainty for each relates to a prospective rather than a current position and has, therefore, to be estimated. D, for example, is the immediate dividend to be received rather than the most recent dividend paid. If uncertainty relates to D, it does so even more to g which is essentially a subjective estimate of long-term growth and even P is liable to unexpected change. Finally, there is the problem of shareholders' tax to consider for this is important to the assessment of the relative cost of retentions and new equity capital. Unless a company has only one shareholder, such relative cost may be impossible to determine because different shareholders will have different tax rates.

Beyond these difficulties problems arise with the notion of profit or wealth maximisation. Most text-book discussions of the cost of capital start off with an acceptance of current market price and current yield levels as a measure of Ke. But what if market price is changing from day to day or even from hour to hour in active trading? It is no good saying Ke can only be estimated at a particular moment in time. This may be true but from a practical point of view on what cost of equity capital should a company's management base its decisions vis-à-vis new investment projects? This brings us to the question of whether management should aim at maximising net present worth or market price, accepting that in the real world the two may diverge considerably and even remain apart for a protracted time. Perhaps it is right to assume that current market price reflects net present worth correctly, but if the expectations of the stock-market and management happen to diverge as to the future growth of the company and its shares (two very different things) what then?[7] One can only conclude that the cost of equity capital is a cost that cannot be determined objectively no matter to what lengths we go. It is not only an opportunity cost but a subjective opportunity cost used by management for determining a cut-off point in capital budgeting. As Weston and Brigham have pointed out, the cost of equity capital Ke, is a required rate of return which necessitates 'very fine judgements' about acceptable rates of return, risk, liquidity and the state of capital markets. 'It would be nice to pretend that these judgements are unnecessary and to specify a precise way of determining the exact cost of equity capital. Unfortunately, this is not possible. Finance is in large part a matter of judgement, and one simply must face this fact.'[8]

THE COST OF DEPRECIATION GENERATED FUNDS

Although a most important item in the calculation of profit, depreciation is an expense which, in any particular year, involves no actual outlay of cash. It is, thus, an expense largely discretionary in nature and greatly dependent for its size on management policy.

The purpose of providing for depreciation in particular years when the wastage of assets may be spread over many, has to do with the desire to show a correct level of annual profit in a continuing enterprise so as to avoid any excessive distribution of dividends and, thus, drain on resources and so as to avoid, also, giving an inaccurate impression of the trend of profits. As a continuing enterprise is one which, by definition, replaces assets that are worn out, the importance of depreciation in conserving resources is immediately apparent.[9] It can be seen, however, that depreciation has much in common with retained earnings and the question arises as to the cost of depreciation generated funds. In other words, for purposes of investment decision making should depreciation be considered 'free' capital or should a cost be assigned to it as a cost is assigned to retained earnings?

Weston and Brigham assert that a charge should be assigned to depreciation generated funds but that that charge is equal to the average cost of capital and can, therefore, be ignored in its calculation.[10]

Although this appears acceptable, it leaves unanswered two important questions. First, if yearly depreciation is an accurate reflection of asset wastage and not, as is so often the case, an overstatement of such wastage, how do we account for the negative growth that, *ceteris paribus*, would accompany the distribution of depreciation funds? And second, how do we account for the fact that depreciation is deductible as an expense in the determination of taxable income?

We have already seen that the cost of capital (Ke) can be stated either in terms of dividends or earnings. Thus:

$$Ke = \frac{De}{Pe} + gt = \frac{Ee}{Pe} + ga$$

What, however, if De is greater than real, as opposed to disclosed, Ee by an amount equal to the depreciation charge? In such a case gx (expansion growth) must become negative. However, gt (total growth) might still be positive if ga (real growth) is sufficient to offset the negative gx. Similarly, where De, Ee and Oe (equity cash flow) are all equal and ga = zero, gx would be negative if the company concerned was the owner of depreciating assets. Were gt to be negative, the cost of capital would actually be *less* than the current dividend yield!

If $De < Ee$ but $Ee = Oe$

$$Ke = \frac{De}{Pe} + gt = \frac{Ee}{Pe} + gt - gx = \frac{Oe}{Pe} + gt - gx$$

Here, Ke would only be less than the current dividend yield if the retention of distributable profits (after corporation tax) was not great enough to compensate for the negative growth caused by no depreciation provision. Clearly, however, it would be absurd to retain funds after tax if the retentions could be included

7 The use of market price as a factor in the cost of capital may appear acceptable with respect to listed companies whose shares are regularly traded at disclosed prices but what of unlisted companies whose shares have no quotation even in over-the-counter markets. The managements of such companies require, no less than those of companies which are quoted, a generally accepted methodology for determining costs of capital.

8 Op. cit. p. 347

9 Space does not allow for a discussion here of the problems associated with the divergence between historical and replacement costs.

10 Op. cit. p. 368

above the line for tax purposes. Indeed, less would need to be retained above the line for the same result because the Receiver, in effect, subsidises the wastage of assets.

If, as is the general rule, $De < Ee < Oe$

$$Ke = \frac{De}{Pe} + gt = \frac{Ee}{Pe} + gt - gx_1 = \frac{Oe}{Pe} + gt - gx_2$$

In this case, the corporate tax factor (q) is still ignored. Because $Ee < Oe$, $gx_1 < gx_2$

But Oe (equity cash flow) = Ee + (Oe — Ee) where Oe — Ee = depreciation

Therefore, $$Ke = \frac{Ee + (Oe - Ee)}{Pe} + gt - gx_2$$

If allowance is then made for the tax deductibility of depreciation, the cost of cash flow (Ke_3) becomes:

$$Ke_3 = \frac{Ee + (Oe - Ee)(1 - q)}{Pe} + gt - gx_2$$

Clearly, the cost of depreciation generated funds *is less* than Ke_1, the cost of retained earnings.

MARGINAL AND AVERAGE COSTS OF CAPITAL AND CAPITAL STRUCTURE

From what has been said above about imputed costs, it is not only the individual costs of capital that concern the appraisal of new investment projects. Theoretically, optimising profits or net present worth requires a minimisation of the average cost of capital and this can only be achieved if the cost of any new funds (i.e. the marginal cost of capital) is less than or at most equal to the average. Looked at comprehensively, a firm's cost of capital becomes the weighted average of the individual component capital costs and it is implicit in the notion that this average can be minimised that an optimum relationship between the individual capital components exists. Space, however, does not permit further elaboration on this important topic.

CONCLUSION

This paper set out to define the cost of capital, to explain why it is important to calculate the cost of capital and to identify the problems associated with its determination. Its finding has been that the cost of capital, like other business costs, represents the minimum return that must be earned by the use of a resource if its employment is to be justified in terms of the profit or wealth maximising goals of an enterprise. In a world of complete certainty, the cost of capital would be equal to the rate of interest (the cost of money determined by the time preferences of users) because in a completely certain world, the problem of different kinds of capital, i.e. loan, preference and equity, would not arise. In an uncertain world, however, different securities need to be ranked according to their respective risks and possibilities of growth and this results in a divergence in the component costs of capital. The reason for calculating the cost of capital, nevertheless, remains. This has to do with determining the desirability of undertaking new investment projects and ranking them in their order of attractiveness. In determining the costs of capital, however, various difficulties arise. In the main, these stem from the fact that the various factors that need to be considered in calculating the cost of capital cannot be known with certainty because they relate to the future. The factors, therefore, have to be estimated and this makes all calculations of the cost of capital unavoidably subjective. This is especially so because the cost of capital has to be related to changing conditions in money and capital markets and hence to actual and prospective changes in interest rates. Further difficulties stem from the fact that interest is treated as an above the line expense in the calculation of corporate taxes, that the income tax of shareholders differs both from individual to individual and between interest and dividend receipts which are usually subject to a measure of tax exemption. Because a company and its shareholders are essentially the same, the cost of equity capital and retained earnings is inextricably bound up with the personal circumstances of shareholders. Finally, the paper noted that it is not only component costs of capital that are important to capital budgeting decisions. It is the average cost of capital, as measured by a weighted mean of component costs, which management should seek to minimise. The paper, however, did not discuss the matter of optimum capital structure (implicit in the notion of a minimum average capital cost) as it was felt that this would involve substantial further discussion beyond the immediate matter under consideration.

Johannesburg
April, 1973

References

Ezra Solomon,
The Theory of Financial Management, Columbia University Press, 1967

J. Fred Weston and Eugene F. Brigham,
Managerial Finance, 3rd Edition, Holt, Rinehart and Winston, 1969

J. Fred Weston and Eugene F. Brigham,
Essentials of Managerial Finance, 2nd Edition, Holt, Rinehart and Winston, 1969

J. Robert Lindsay and Arbold W. Sametz,
Financial Management: An Analytical Approach, Revised Edition, Richard D. Irwin Inc., 1967